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The Plant Location and Flexible Technology Acquisition Problem

Abstract

In many industries, production-distribution networks have become more complex due to globalization. In particular, increasing interdependencies among the structural decisions call for the development of integrated models. In this paper, we present a mathematical model for simultaneous optimization of the plant location, capacity acquisition and technology selection decisions in a multi-commodity environment. The proposed model represents the possible scale and scope economies associated with manufacturing technology alternatives. The problem is formulated as a mixed integer nonlinear program with concave costs. We developed an exact and three heuristic solution procedures. Using these procedures, we are able to solve fairly large facility design problems with reasonable computational effort.

Keywords: Facilities design and planning, location, flexible manufacturing systems, production-distribution systems, concave minimization.

Localisation d'usine et le Problème de Saisie de Technologie Flexible

Résumé

Dans beaucoup d'industries, les réseaux de production-distribution est devenu plus complexe a cause de globalization. En particulier, les interdépendances croissantes parmi les décisions structurales nécessitent le développement des modèles intégrés. Dans cet article, nous présentons un modèle mathématique pour l'optimisation simultanée de localisation d'usine, de la saisie de capacité et de la sélection de technologie dans un environnement de multi-produits. Le modèle proposé représente les économies d'échelle et de portée associées aux differents technologies. Le problème est formulé comme un programme non linéaire de nombre entier avec des coûts concaves. Nous avons développé des procédures exactes et trois heuristiques de solution. En utilisant ces procédures, nous pouvons résoudre des problèmes de grand tallies avec un effort de calcul raisonnable.

Mots-clés: Localisation, technology flexible, systèmes de production-distribution, minimisation concave.

1 Introduction

The complexity of a firm's production-distribution network increases with the level of globalization. In particular, coordination of the firm's operations throughout the entire system of facilities around the world becomes more challenging (McGrath and Hoole, 1998). Furthermore, the structural decisions at each plant (i.e. location, size, technology content and product mix) become highly interdependent. One of the reasons for this phenomenon is the subsidized financing and preferential tax rates provided by governments to attract investments in their jurisdiction. Altshuler et al. (1998) reported that the manufacturing facility location decisions of U.S. multinational corporations are becoming increasingly sensitive to tax rate differences across countries. In many cases, there are also scale economies associated with the available incentives, for example the investment tax credits for manufacturing facilities in Atlantic Canada increases more than proportionately by the total investment. Another factor that fosters the interaction between facility design decisions is the locational variations in the technology acquisition and operation costs. For example, a major player in the telecommunications industry found out that the technology cost of establishing a 15,000 unit facility (for the production of a printed circuit board assembly) in Malaysia is only 24% of the cost of a similar facility in North Carolina, U.S.

In this paper, we provide an integrated model for simultaneous optimization of the facility location, capacity acquisition and technology selection decisions. Our approach would be particularly useful when the interactions between these facility design decisions are profound e.g., in global manufacturing companies. The proposed model constitutes an extension of the multi-commodity plant location problem. At each site, there is a set of product-dedicated technology alternatives for each commodity as well as a set of alternative flexible technologies each capable of manufacturing the entire product range. Typically, product-dedicated technologies present an opportunity for achieving scale economies (Luss, 1982), whereas flexible manufacturing systems offer economies of scope (Talaysum et al., 1986). Our model facilitates the analysis of this difficult trade-off within the context of facility design decisions. The aim is to identify the minimum cost configuration of a single echelon of manufacturing plants so as to satisfy the market demand.

The remainder of the article is organized as follows: A selective review of the related literature is provided in Section 2. Problem definition, mathematical formulation, and analytical properties are presented in Section 3. In Section 4 and 5 the exact and heuristic procedures are outlined. Computational results are reported in Section 6 and Section 7 concludes the article with several remarks.

2 Literature Review

Earlier studies on multi-commodity location problems constitute extensions of the methodology for the simple facility location problem. Warszawski (1973), Karkazis and Boffey (1981), and Neebe and Khumawala (1981) proposed models which incorporate fixed costs associated with the product-plant assignments in addition to the fixed plant set up costs and linear transportation costs. Klincewicz and Luss (1987) presented a dual-based approach for this problem. In their seminal paper, Geoffrion and Graves (1974) analyzed the problem of locating distribution centers (DCs), which would be used in serving customers demand from a set of existing plants. They suggested the use of Benders' decomposition. Their work was extended by Moon (1989), whose model incorporates economies of scale in DC operation costs. More recently, Pirkul and Jayaraman (1996) provided another extension of Geoffrion and Graves (1974) by incorporating the plant location decisions as well. A recent review of the state of the art in production-distribution system design can be found in Vidal and Goetschalckx (1997).

The integrated facility design model presented in this paper incorporates not only the plant location but also capacity acquisition and technology selection decisions. During the past decade, the methodology on capacity expansion evolved in three directions: Integration with production planning and inventory control (e.g., Sethi et al., 1995), incorporation of equipment replacement decisions (e.g., Rajagopalan, 1998), and incorporation of technology selection decisions. The prevailing studies in the third group focus on analyzing the trade-offs between product-dedicated technologies and flexible manufacturing systems, and hence they are relevant to our work.

Burstein (1988) developed a mixed integer mathematical model to generate decision rules that

are used to analyze several hypotheses about the introduction of flexible and/or dedicated automation. Fine and Freund (1990) formulated the a two-stage stochastic program that maximizes the expected profit. The first stage makes the technology selection decision and the second stage optimizes the production decisions after the realization of the market demand. The authors also characterized the necessary and sufficient conditions to invest in flexible technology and analyzed the sensitivity of the investment decisions to changing level of risk and demand distributions. Rajagopalan (1993) proposed a multi-period model that prescribes the capacity decisions so as to minimize total investment costs. The model is based on the assumption that the demand is deterministic and nondecreasing. As it is formulated, the problem transforms to the simple facility location problem, for which various efficient solution procedures exist. Li and Tirupati (1994) further incorporated economies of scale and arbitrary demand structures. They developed several heuristics that provide quite good expansion schedules. Overall, the consensus is that, although more expensive, flexible technology could be an attractive alternative to hedge against future fluctuations in demand or to gain from economies of scope.

It is important to note that facility location models, in general, do not incorporate the capacity acquisition and technology selection decisions explicitly. (A notable exception is the model of Klincewicz et al. (1988) which considers dynamic location of DCs and their capacity planning in a single product dynamic two echelon production-distribution system.) Likewise, capacity expansion and technology selection models typically do not consider location decisions. The model in the next Section constitutes a step towards establishing the link between these strategic design decisions.

3 The Problem

In this paper, we use the term *plant* in referring to a manufacturing establishment to be constructed at one of the alternative sites. A plant may contain a number of *dedicated facilities* each capable of producing a single commodity, and/or a *flexible facility* that can produce any subset of products. Given a set of alternative plant locations, a set of products to be manufactured, a

set of dedicated technology alternatives for each product, and a set of flexible technology alternatives, *the plant location and flexible technology acquisition problem* involves optimization of the location, capacity, and technology decisions so as to serve the market demand with minimum cost. Assuming that there are no *a priori* limitations on the capacity to be acquired at each facility, we study the uncapacitated plant location and flexible technology acquisition problem (UPL&FTAP).

Now, we turn to the notation that will be used in developing the mathematical formulation of the problem:

I	:	Set of alternative plant locations, indexed by i, $\|I\| = m$,
J	:	Set of customer zones, indexed by j, $\|J\| = n$,
P	:	Set of products, indexed by p, $\|P\| = r$,
H_p	:	Set of alternative technologies for product $p \in P$, indexed by h_p, and,
H	:	Set of alternative flexible technologies, indexed by h.

Let $f_{h_p i}(.)$ represent the cost of acquisition and operation of technology h_p at plant i as a function of the amount of capacity to be built-in, and $f_{hi}(.)$ represent that of technology h. For the ease of exposition of the model, we will take advantage of the conditional dominance property presented in Verter (1997): At the optimal solution each facility will adopt a single technology that varies with its size. In effect, each facility faces the lower envelope of the associated cost functions as the effective cost of capacity acquisition and operation (see Figure 1). Thus, $r+1$ functions are sufficient to represent all the technology cost information at each plant:

$$f_{ip}(.) = \min_{h_p \in H_p} \{f_{h_p i}(.)\}, \ \ \forall\, (i,p), \ \text{ and, } f_i(.) = \min_{h \in H} \{f_{hi}(.)\}, \ \ \forall\, i. \tag{1}$$

Since the product-dedicated technologies present scale economies and flexible technologies present economies of scope, $f_{ip}(.)$ and $f_i(.)$ are monotone increasing concave functions. In general, finding closed-form expressions for these functions would be a formidable task. Nevertheless, the proposed solution algorithm does not require closed-form characterizations for $f_{ip}(.)$ and $f_i(.)$. Let,

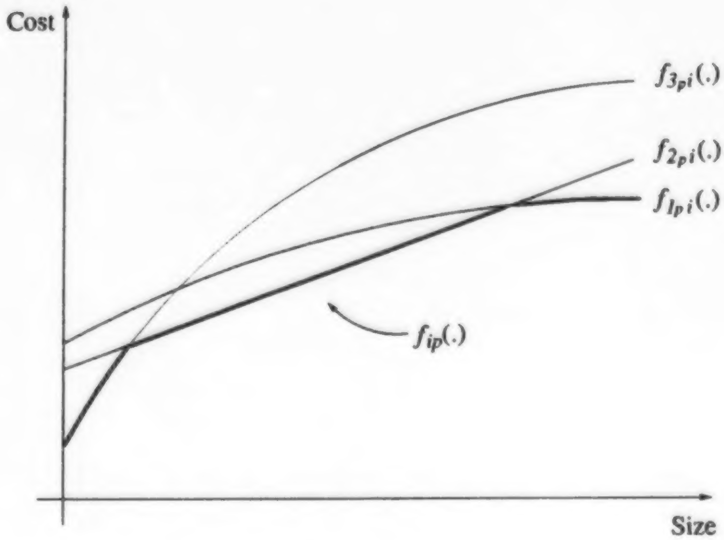

Figure 1: Lower envelope of technology acquisition and operation cost of a dedicated technology.

Y_i	:	binary decision variable for opening plant i,
x^D_{ijp}	:	continuous decision variable for the amount of product p produced by a dedicated technology at plant i and shipped to customer zone j,
x^F_{ijp}	:	continuous decision variable for the amount of product p produced by a flexible technology at plant i and shipped to customer zone j,
F_i	:	the fixed cost of opening a plant at site i,
c_{ijp}	:	the unit cost of shipping product p from plant i to customer zone j,
D_{jp}	:	the demand of customer zone j for product p.

We propose the following model for UPL&FTAP:

$$\begin{aligned}
\text{minimize } z \;=\;& \sum_{i\in I} F_i Y_i + \sum_{i\in I} f_i\Big(\sum_{j\in J}\sum_{p\in P} x^F_{ijp}\Big) + \sum_{i\in I}\sum_{p\in P} f_{ip}\Big(\sum_{j\in J} x^D_{ijp}\Big) \\
& + \sum_{i\in I}\sum_{j\in J}\sum_{p\in P} c_{ijp}(x^F_{ijp} + x^D_{ijp}) && (2)\\
\text{s.t.}\quad & \sum_{i\in I} x^F_{ijp} + x^D_{ijp} = D_{jp}, \;\; \forall\,(j,p), && (3)\\
& x^F_{ijp} \le D_{jp} Y_i, \;\; \forall\,(i,j,p), && (4)\\
& x^D_{ijp} \le D_{jp} Y_i, \;\; \forall\,(i,j,p), && (5)\\
& x^F_{ijp}, x^D_{ijp} \ge 0, \;\; \forall\,(i,j,p), && (6)\\
& Y_i \in \{0,1\}, \;\; \forall\, i. && (7)
\end{aligned}$$

Constraints (3) guarantee that every customer zone's demand is satisfied. Constraints (4) and (5) ensure that only open plants can produce and ship products. Finally, (6) and (7) are non-negativity and binary restrictions over the decision variables. In this formulation, the capacity to be built at a facility is determined by the total production assigned to that facility at the optimal solution.

UPL&FTAP is a single period, single echelon, and deterministic problem which can be reduced to the uncapacitated facility location problem (UFLP) when there is a single product with linear technology costs. Krarup and Pruzan (1983) showed that UFLP is NP-Complete, hence UPL&FTAP is also NP-Complete.

UPL&FTAP can be cast as a single source minimum concave cost layered network flow problem (MCNFP), as in the instance depicted in Figure 2. Therefore, the techniques available for solving MCNFP are also suitable for solving UPL&FTAP. A local optimal solution is not necessarily the global optimum for MCNFP, because the objective function is concave. Nevertheless, the global optimum is an extreme flow. Most of the algorithms for MCNFP take advantage of this property. Guisewite and Pardalos (1990) grouped the prevailing algorithms into three categories; branch and bound, dynamic programming and extreme point ranking. It is important that the variety of techniques applicable for solving MCNFP are for more general networks and they do not exploit the structural properties of UPL&FTAP.

In the remainder of this Section, we will focus on the properties of UPL&TAP, which are instrumental in developing a solution procedure. One such property is that, at the optimum solution the demand at customer zone j for product p will be fully served by a single facility. This property, in reference to MCNFP, is proven in Zangwill (1968). It enables us to characterize the set of alternative sizes for each facility.

Proposition 1 *The number of alternative sizes for a dedicated facility is* 2^n *and that of a flexible facility is* 2^{nr}.

The proof is based on the observation that, at the optimum solution, each dedicated facility

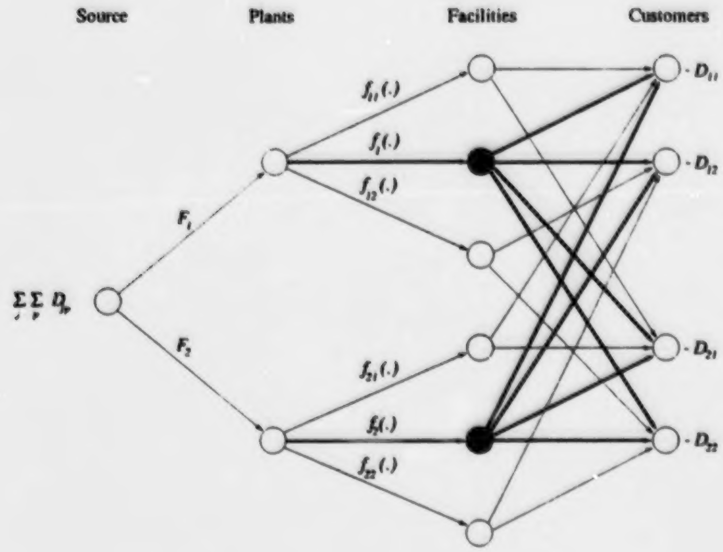

Figure 2: MCNFP representation of UPL&FTAP for $m = 2, n = 2, r = 2$

can fully serve all possible combinations of the customer zones for the associated product, and each flexible facility can fully serve all possible combinations of customer zone-product pairs. In effect, although we use continuous functions $f_{hi}(.)$ and $f_i(.)$ in representing capacity costs in the model, they need to be evaluated only at a finite number of capacity alternatives. The following proposition is important for characterizing technology content of the plants.

Proposition 2 *At the optimum solution, either a dedicated or a flexible technology (but not both) will be used in manufacturing product p at plant i.*

Proof: Please see the Appendix.

4 An Algorithm for Solving UPL&FTAP

The algorithm devised here adopts the *progressive piecewise linear underestimation* (PPLU) technique developed by Verter and Dincer (1995). Each iteration of PPLU involves construction of a new subproblem on the basis of the current piecewise linear underestimate for the total cost

function. The solution of this subproblem is then used for improving the underestimate so as to facilitate the next iteration. The optimal solution to a subproblem corresponds to an extreme flow of the underlying network of UPL&FTAP. Since the number of extreme flows is finite, the proposed algorithm finds the optimal solution in a finite number of iterations. In the worst case, the algorithm would visit all the extreme points. The following proposition gives bounds on the number of extreme points.

Proposition 3 *The lower and upper bounds on the number extreme points are* $2m^{nr}$ *and* $(2m)^{nr}$ *respectively.*

Proof: Please see the Appendix.

Note that flexible technology costs are separable in terms of plants and dedicated technology costs are separable in terms of facilities. Naturally, the underestimate functions posses the same property. Let D_p represent the total demand for product p and D represent the total demand for all products (i.e., $D_p = \sum_j D_{jp}$ and $D = \sum_p D_p$). We use the straight line connecting the origin to $(D_p, f_{ip}(D_p))$ as the initial underestimate at dedicated facility (i,p). Similarly, the straight line passing from the origin and $(D, f_i(D))$ is used at flexible facility i to initialize the algorithm. At an iteration of PPLU, a pseudo-facility is associated with each segment of the current piecewise linear underestimate. Let,

K_{ip}	:	the set of pseudo-facilities of facility (i,p), indexed by k, $\|K_{ip}\| = k_{ip}$,
K_i	:	the set of flexible pseudo-facilities of plant i, indexed by k, $\|K_i\| = k_i$,
F_{ipk}	:	the fixed cost of opening pseudo-facility k of facility (i,p),
F_{ik}	:	the fixed cost of opening flexible pseudo-facility k of plant i,
c_{ipk}	:	the unit cost of technology acquisition at pseudo-facility k of facility (i,p),
c_{ik}	:	the unit cost of technology acquisition at flexible pseudo-facility k of plant i,
R_{ipk}	:	the lower bound on the size of pseudo-facility k of facility (i,p),
R_{ipk+1}	:	the upper bound on the size of pseudo-facility k of facility (i,p).
R_{ik}	:	the lower bound on the size of flexible pseudo-facility k of plant i, and,
R_{ik+1}	:	the upper bound on the size of flexible pseudo-facility k of plant i.

The associated subproblem can be modeled as follows:

$$\begin{aligned}
\text{minimize } z_{SP1} = \quad & \sum_{i\in I} F_i Y_i + \sum_{i\in I}\sum_{k\in K_i} F_{ik} Y_{ik} + \sum_{i\in I}\sum_{p\in P}\sum_{k\in K_{ip}} F_{ipk} Y_{ipk} \\
& + \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{k\in K_i} c^F_{ijpk} x^F_{ijpk} + \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{k\in K_{ip}} c^D_{ijpk} x^D_{ijpk} && (8) \\
\text{s.t.} \quad & \sum_{i\in I}\sum_{k\in K_i} x^F_{ijpk} + \sum_{i\in I}\sum_{k\in K_{ip}} x^D_{ijpk} = D_{jp}, \ \forall\,(j,p), && (9) \\
& x^F_{ijpk} \le D_{jp} Y_{ik}, \ \forall\,(i,j,p),\ k\in K_i, && (10) \\
& x^D_{ijpk} \le D_{jp} Y_{ipk}, \ \forall\,(i,j,p),\ k\in K_{ip}, && (11) \\
& Y_{ik} \le Y_i, \ \forall\, i,\ k\in K_i, && (12) \\
& Y_{ipk} \le Y_i, \ \forall\,(i,p),\ k\in K_{ip}, && (13) \\
& R_{ik} Y_{ik} \le \sum_{j\in J}\sum_{p\in P} x^F_{ijpk} \le R_{ik+1} Y_{ik}, \ \forall\, i,\ k\in K_i, && (14) \\
& R_{ipk} Y_{ipk} \le \sum_{j\in J} x^D_{ijpk} \le R_{ipk+1} Y_{ipk}, \ \forall\,(i,p),\ k\in K_{ip}, && (15) \\
& x^F_{ijpk}, x^D_{ijpk} \ge 0, \ \forall\,(i,j,p,k), && (16) \\
& Y_i, Y_{ik}, Y_{ipk} \in \{0,1\}, \ \forall\,(i,p,k), && (17)
\end{aligned}$$

where

$$c^F_{ijpk} = c_{ijp} + c_{ik}, \ \text{ and, } c^D_{ijpk} = c_{ijp} + c_{ipk}$$

and the additional decision variables are:

Y_{ik} : binary variable for opening flexible pseudo-facility k of plant i,

x^F_{ijpk} : the quantity of product p shipped from flexible pseudo-facility k of plant i to customer zone j.

Y_{ipk} : binary variable for opening pseudo-facility k of facility (i,p),

x^D_{ijpk} : the quantity shipped from pseudo-facility k of facility (i,p) to customer zone j.

Constraints (9) ensure that demand will be fully satisfied, whereas (10) and (11) guarantee that customers receive shipments only from open pseudo-facilities and (12) and (13) specify that a pseudo-facility can be opened only if its plant is open. Constraints (14) and (15) ensure that total production of each pseudo-facility is between its lower and upper bounds. Finally (16) and (17) are non-negativity and binary restrictions over the decision variables. The formulation will open the pseudo-facility that corresponds to the size range containing the optimal size of that facility because, the piecewise underestimates are also a concave functions. Therefore,

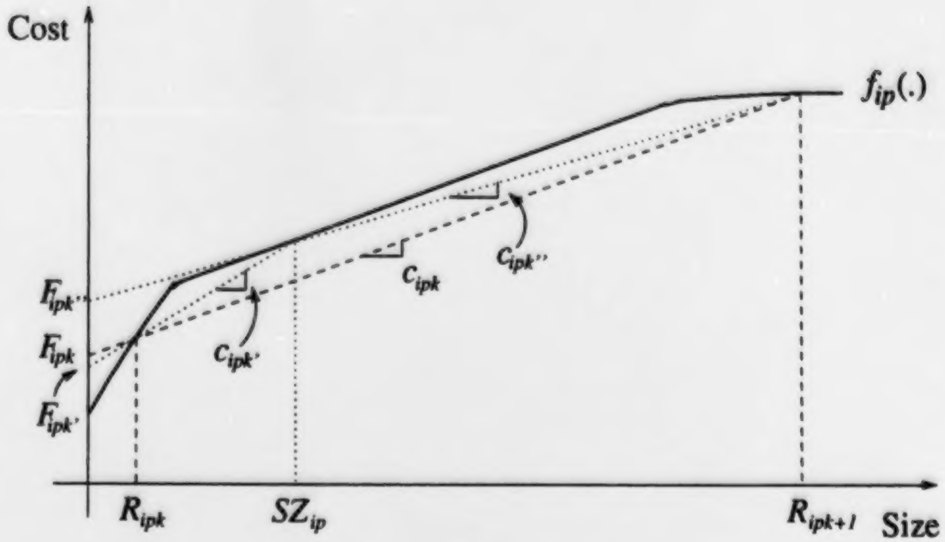

Figure 3: Improving the underestimation

constraints (14) and (15) can be omitted from the formulation.

Figure 3 depicts utilization of the solution to a subproblem in improving the current underestimate for a dedicated technology at a plant. Let SZ_{ip} denote the size of facility (i,p) at an iteration of the algorithm i.e., $SZ_{ip} = \sum_j \sum_k x_{ijpk}^{D*}$, where x_{ijpk}^{D*} is obtained from (8)-(17). If SZ_{ip} is equal to one of the interval endpoints (i.e., R_{ipk}) then the cost of acquiring dedicated technology for product p at plant i is exactly represented by the underestimate, hence no new pseudo-facility is generated. Otherwise, the pseudo-facility k of (i,p) is replaced by two new pseudo-facilities, which amounts to an improvement in the approximation of technology costs. This procedure is also used in improving the current underestimate for flexible technology acquisition costs. The following constitutes a formal statement of the algorithm. The reader is referred to Verter and Dincer (1995) for a more detailed description of the PPLU technique.

```
Step 1:  Initialize:  Calculate the parameters of the first subproblem,
         K_i = K_ip = {1}
Step 2:  Solve the arising subproblem.  Get Y_i^*, Y_ik^*, Y_ipk^*, x_ijpk^F*, x_ijpk^D*.
Step 3:  If there is no need for improving the current underestimate,
               Set x_ijp^F* = Σ_k x_ijpk^F*, x_ijp^D* = Σ_k x_ijpk^D*, STOP.
         Else modify K_i, K_ip and compute the parameters for the new
               subproblem.  Go to Step 2.
```

It is possible to improve the formulation of the subproblem by defining the flow variables as fractions rather than quantities. The reformulation proved useful for computational purposes. Let,

$$X_{ijpk}^F = x_{ijpk}^F / D_{jp}, \;\; X_{ijpk}^D = x_{ijpk}^D / D_{jp}, \;\; C_{ijpk}^F = (c_{ijpk}^F) D_{jp}, \;\; C_{ijpk}^D = (c_{ijpk}^D) D_{jp}.$$

We can reformulate the subproblem as follows:

$$\begin{aligned}
\text{minimize } z_{SP2} = \quad & \sum_{i \in I} F_i Y_i + \sum_{i \in I} \sum_{k \in K_i} F_{ik} Y_{ik} + \sum_{i \in I} \sum_{p \in P} \sum_{k \in K_{ip}} F_{ipk} Y_{ipk} \\
& + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \sum_{k \in K_i} C_{ijpk}^F X_{ijpk}^F + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \sum_{k \in K_{ip}} C_{ijpk}^D X_{ijpk}^D && (18) \\
\text{s.t.} \quad & \sum_{i \in I} \sum_{k \in K_i} X_{ijpk}^F + \sum_{i \in I} \sum_{k \in K_{ip}} X_{ijpk}^D = 1, \;\; \forall\, (j,p), && (19) \\
& X_{ijpk}^F \le Y_{ik}, \;\; \forall\, (i,j,p), \; k \in K_i, && (20) \\
& X_{ijpk}^D \le Y_{ipk}, \;\; \forall\, (i,j,p), \; k \in K_{ip}, && (21) \\
& Y_{ik} \le Y_i, \;\; \forall\, i, \; k \in K_i, && (22) \\
& Y_{ipk} \le Y_i, \;\; \forall\, (i,p), \; k \in K_{ip}, && (23) \\
& X_{ijpk}^F, X_{ijpk}^D \ge 0, \;\; \forall\, (i,j,p,k), && (24) \\
& Y_i, Y_{ik}, Y_{ipk} \in \{0,1\}, \;\; \forall\, (i,p,k), && (25)
\end{aligned}$$

The following proposition enables us to benchmark the performance of the algorithm we devised:

Proposition 4 *The subproblem is a special case of the two-echelon uncapacitated facility location problem* (TUFLP).

The proof is outlined in the Appendix. Gao and Robinson (1992) provides an efficient dual based branch and bound algorithm for TUFLP. The efficiency of a dual based algorithm depends

on the possibility of obtaining tight lower bounds from the LP relaxation. Our preliminary computational experiments, however, showed that LP-relaxation of (18)-(25) does not provide tight lower bounds. In an effort to improve the LP bound we added the following constraints to the subproblem:

$$\sum_{k \in K_{ip}} Y_{ipk} + \sum_{k \in K_{ip}} X^F_{ijpk} \leq Y_i, \quad \forall\, (i,j,p), \tag{26}$$

which ensure that a product cannot be produced by a dedicated and a flexible technology at the same plant. Note that (26) is a restatement of Proposition 2. Subproblem(18)-(26) is amenable to solution by the use of LP-based branch and bound. Note also that we no longer need Constraint (23) in the formulation. After this transformation Step 2 of the algorithm changes as follows:

`Step 2.0:`	`Transform the parameters, Find` C^F_{ijpk}, C^D_{ijpk}.
`Step 2.1:`	`Solve the arising subproblem (get` $Y_i^*, Y_{ik}^*, Y_{ipk}^*, X^{F*}_{ijpk}, X^{D*}_{ijpk}$`).`
`Step 2.2:`	`Transform the variables, Find` $x^{F*}_{ijpk}, x^{D*}_{ijpk}$.

The value of the original objective function (2) at any solution given by the subproblem constitutes an upper bound on the optimum solution of UPL&FTAP, whereas z_{SP_2} constitutes a lower bound. The gap between these bounds can be used as an indicator of the quality of the solutions at each iteration. Therefore, it is possible to devise an ϵ-heuristic. Naturally, such a heuristic would require less iterations than the exact procedure. Note also that, at each iteration of the PPLU technique, the subproblems are solved to optimality. An alternative heuristic scheme can be developed by using non-exact procedures for solving the subproblems. The next Section describes three such heuristic procedures.

5 UFLP Based Heuristics

In our heuristic scheme the subproblems are solved approximately and the algorithm stops when no more improvement of the underestimate is necessary. Note that should our heuristic produce non-extreme solutions to the arising subproblems, there would be no guarantee that the

PPLU algorithm would stop. Furthermore, the heuristic should avoid solutions which violate Proposition 2.

Given a set of open plants, the subproblem reduces to an instance of UFLP (see Figure 4) with fixed charges of F_{ik}'s and F_{ipk}'s, and variable costs of C^F_{ijpk}'s and C^D_{ijpk}'s. The proposed heuristic algorithms start with all plants open. The arising UFLP is solved by Erlenkotter's (1978) dual based algorithm. Then, our algorithm sequentially checks the plants to detect any violation of Proposition 2 and eliminates them: If a product is assigned to both dedicated and flexible facilities at a plant, the facility with smaller total cost is assigned to be the sole producer of the product. Given the product, technology and flow information, choosing the best subset of open plants is a hard combinatorial problem by itself and hence we devised a drop procedure: A plant is closed if there is no product assigned to it, or the savings associated with the closure decisions (i.e., fixed costs and economies of scale from the other plants) surpasses the resulting additional transportation costs. When a plant is closed all of its customers are transferred to other plants. These plants are chosen according to a cost analysis which tries to find the best plant, for a product, that gives the lowest additional transportation and technology cost. After this stage the three heuristics follow different steps: The first heuristic iterates the drop procedure and stops when there is no more plants to close. Based on this set of open plants the second heuristic, however, solves another UFLP to improve the assignments. The third heuristic solves a UFLP after each iteration of the drop procedure

Starting with the first heuristic (H1), let $I^+ \subseteq I$ for which $Y_i = 1, \ \forall \, i \in I^+$. The following segment replaces Step 2.1 in the PPLU algorithm outlined before:

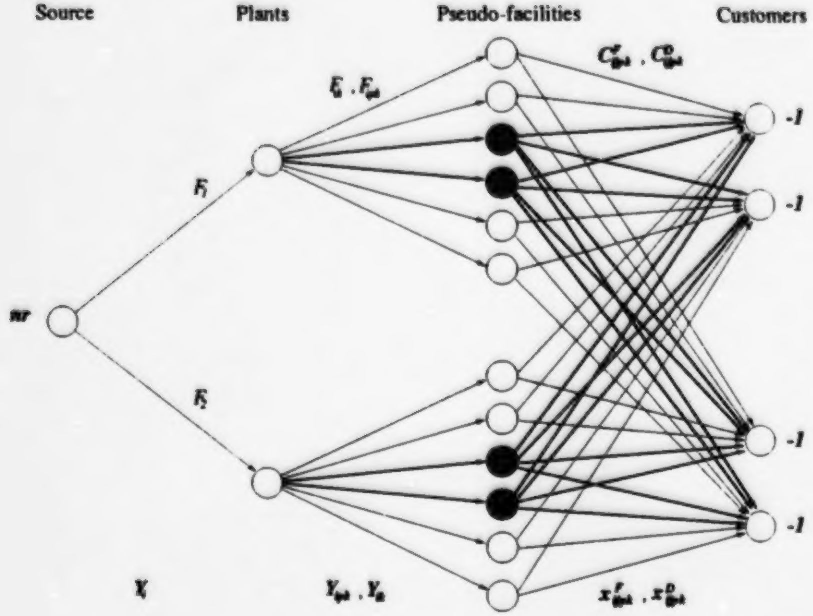

Figure 4: MCNFP representation of the subproblem for $m = 2, n = 2, r = 2$.

Step 2.1.0:	**Set $I^+ = I$.**
Step 2.1.1:	**Solve the arising UFLP (get $Y^*_{ik}, Y^*_{ipk}, X^{F*}_{ijpk}, X^{D*}_{ijpk}$).**
Step 2.1.2:	**If $\exists(i,j,p)$ for which $\sum_{k\in K_{ip}} Y^*_{ipk} + \sum_{k\in K_{ip}} X^{F*}_{ijpk} > 1$, then Eliminate either the flexible or dedicated whichever results in smaller penalty. Modify the variables.**
Step 2.1.3:	**Check if $\exists i$ such that $Y^*_{ik} = 0 \; \forall \, k \in K_i$, $Y^*_{ipk} = 0 \; \forall \, p \in P, \, k \in K_{ip}$, Set $I^+ = I^+ \setminus \{i\}$.**
Step 2.1.4:	**For all $i \in I^+$ Compute the savings from closing plant i,**
Step 2.1.5:	**Subtract i that has the largest positive savings from I^+, Assign its customers to the least cost facility at open plants.**
Step 2.1.6:	**If I^+ is modified, then go to Step 2.1.4, Else STOP with $(Y^*_i, Y^*_{ik}, Y^*_{ipk}, X^{F*}_{ijpk}, X^{D*}_{ijpk})$.**

The second heuristic (H2) differs from the first one only in Step 2.1.6, which is modified as follows:

Step 2.1.6:	**If I^+ is modified, then go to Step 2.1.4, Else perform Steps 2.1.1-3. STOP with $(Y^*_i, Y^*_{ik}, Y^*_{ipk}, X^{F*}_{ijpk}, X^{D*}_{ijpk})$.**

The last step for the third heuristic (H3) is as follows:

```
Step 2.1.6:  If I+ is modified, then go to Step 2.1.1,
             Else STOP with (Y*_i, Y*_ik, Y*_ipk, X^F*_ijpk, X^D*_ijpk).
```

These heuristics could be improved by the use of more elaborate plant selection procedures, such as pairwise or three-way interchanges. Nevertheless, as we demonstrate in the next Section, the proposed heuristic scheme is quite effective in solving the subproblems.

6 Computational Results

In this Section, we report on our computational experiments which enabled us to observe the computational performance of the proposed solution procedures. We implemented all of the algorithms for real number arithmetic in C, with the exception of the dual based procedure of Erlenkotter (1978) which is used to solve the arising UFLPs. This code, originally implemented in FORTRAN, was obtained from Klincewicz and Luss (1987). All the computations were carried out on a SUN Sparc 10.

Solving subproblems is computationally the hardest part of our solution algorithm. Therefore, from the overall computational performance standpoint, it is crucial to solve the arising subproblems efficiently. To this end, an LP-based branch and bound algorithm is coded in C using CPLEX subroutines. We disabled the preprocessing and utilized dual simplex in solving LPs in the implementation. Also 'Best Estimate Search' was used in node selection, and 'variable with the maximum infeasibilty' was used in variable selection. The rest of the parameters were kept at their default values.

The experiments are based on two sets of test problems that we developed from two well known UFLPs (*cap71*, and *cap131*) provided by Kuehn and Hamburger (1963). *Cap71* is a 16 location 50 customer zone problem, whereas *cap131* has 50 locations and 50 customer zones. Both problems are available at the OR-Library (Beasley, 1990). We generated their multicommodity versions

h	$\tilde{\beta}$	$\tilde{\alpha}$
1	[45,50]	[0.65,0.70]
2	[22,28]	[0.72,0.77]
3	[12,18]	[0.79,0.84]
	$\tilde{a}$	$\tilde{b}$
4	0	[2.5,3.5]
5	[4000,5000]	[1.5,2.5]

Table 1: Uniform Distribution Ranges for Cost Parameters.

by the use of uniform distributed transportation costs and demand figures for five products. Each new parameter is drawn from a uniform distribution with endpoints 20% below and above the value of the associated parameter in the original problem. The resulting correlation between products is justifiable because population and distance are the major determinants of demands and transportation costs respectively. Plant fixed costs are kept constant at, $F_i = \$75,000$ in all instances.

We further augmented the problems by assuming five dedicated technology alternatives for each plant-product pair and five flexible technology alternatives at each site. Technologies 1,2, and 3 represent scale/scope economies, whereas Technologies 4 and 5 have a fixed charge linear cost structure. The validity of these functions in modeling technology costs is established by Luss (1982). Note that the number of technology alternatives and their shapes do not pose additional complexity to the problem because, our procedures do not need to find a closed form expression for their lower envelopes. (They do, however, require technology costs to be concave.) Our technology acquisition and operation costs have the following forms:

$$
\begin{aligned}
f_{h_p i}(.) &= \tilde{\beta}(.)^{\tilde{\alpha}}, \quad h = 1,2,3, \ \ i = 1,\ldots,16, \ \ p = 1,\ldots,5, \\
f_{h_p i}(.) &= \tilde{a} + \tilde{b}(.), \quad h = 4,5, \ \ i = 1,\ldots,16, \ \ p = 1,\ldots,5, \\
f_{h_i}(.) &= (1.4)\tilde{\beta}(.)^{\tilde{\alpha}}, \quad h = 1,2,3, \ \ i = 1,\ldots,16, \\
f_{h_i}(.) &= (1.4)\tilde{a} + (1.4)\tilde{b}(.), \quad h = 4,5, \ \ i = 1,\ldots,16,
\end{aligned}
$$

where $\tilde{\alpha}, \tilde{\beta}, \tilde{a}$, and $\tilde{b}$ are uniform distributed random variables. Table 1 depicts the ranges of the

Number of Iterations	Optimal	(16,50,5) H1	H2	H3	(50,50,5) H1	H2
4					19	1
5		1			59	26
6		10	1	4	50	58
7	8	20	10	8	39	61
8	19	35	21	17	17	28
9	40	38	46	27	10	15
10	49	30	28	46	4	7
11	32	35	25	34	1	3
12	24	20	34	30		1
13	13	5	19	21	1	
14	12	5	10	9		
15	2	1	5	2		
16						
17	1		1	2		

Table 2: Number of PPLU Iterations Required

uniform distributions used for generating the technology cost function parameters. We assumed that flexible technologies are on the average 40% more expensive. This is compatible with the figures used in technology selection literature. See, for example, Rajagopalan (1993), Li and Tirupati (1994), and Lim and Kim (1998). The uniform distribution ranges are chosen so that, when all cost parameters take their average values, each technology alternative would require about the same investment for a 5,000 unit facility. This enables us to avoid the trivial cases, where linear technology costs constitute the lower envelope.

Tables 2 and 3 present the results of our computational experiments on two sets of problems. The first set contains 200 problems with 16 sites, 50 customer-zones, and five products (based on *cap71*), represented as (16,50,5). The problems in this set are solved both optimally and approximately by all three heuristics. With the same notation, the second set of 200 problems (generated from *cap131*) are represented as (50,50,5). They are solved only with the first two heuristic procedures.

		(16,50,5)			(50,50,5)	
	Optimal	H1	H2	H3	H1	H2
# Iterations						
AVG	10.4	9.5	10.5	10.6	6.2	6.9
MIN	7	5	6	6	4	4
MAX	17	15	17	17	13	12
CPU Time (sec)						
AVG	495.6	16.2	22.5	78.0	65.2	69.2
MIN	52.9	5.8	9.5	30.5	35.6	35.8
MAX	1856.3	31.0	49.3	171.2	160.9	115.3
Inaccuracy (%)						
AVG		0.72	0.26	0.26	3.11	2.62
MIN		0.08	0.00	0.00	0.01	0.00
MAX		1.86	0.98	1.20	6.78	5.59

Table 3: Summary of Computational Results

Table 2 reports on the computational requirements of the proposed algorithms. For example, optimal solution of 49 of the smaller test problems took 10 PPLU iterations. It is important to note that none of the 200 problems in the first set took more than 17 iterations to solve optimally. Given that the PPLU algorithm could take at least 16^{250} iterations for these problems in the worst case, the average computational performance of the proposed algorithm is quite encouraging. Table 3 provides a summary of our computational results. Note that there is no significant difference between the heuristic scheme and the exact solution procedure in terms of the number of iterations required. This is also evident from Table 2, where the four columns associated with the (16,50,5) problems present quite similar bell-shaped patterns. Thus, the order of magnitude difference between the CPU time requirements of the heuristic and exact solution procedures is primarily due to the efficiency of the former in solving the arising sub-problems. In fact, it was the excessive computational requirements of the sub-problems that prevented us from finding exact solutions to the (50,50,5) problems.

In Table 3 , we also provide summary statistics about the inaccuracy of the proposed heuristic algorithms. Inaccuracy was calculated as the percent deviation from the optimal solution for

the first set of problems, whereas we reported the percent deviation from the best lower bound (produced by the last piecewise linear underestimate) for the larger problems. In solving the (16,50,5) problems, H3 required three times more computational time than H2 without a significant gain in the accuracy of the solution. Thus, we focused on the first two heuristics in tackling the (50,50,5) UPL&FTAPs: On the average, H2 required 6% more CPU time than H1 for solving the larger problems and this led to a 15% improvement in the quality of the solution. In fact, the second heuristic identified the optimal solution for 10 problems in the first set and 8 problems in the second set, whereas H1 was not able to find any exact solutions. Another interesting result of our computational results is that the heuristic scheme seems to require less PPLU iterations for larger problems.

Each (16,50,5) UPL&FTAP can be cast as a single source four-layer MCNFP (see Figure 2) with 363 nodes, whereas there are 601 nodes in the equivalent MCNFP formulation of a (50,50,5) problem. The largest sub-problems that arise during the solution of these two sets of UPL&FTAPs, on the average, require the formulation of 470 and 650 node network models (see Figure 4) respectively. These problems are considerably larger than the ones solved in the recent papers on MCNFP e.g., Amiri and Pirkul (1997) and Balakrishnan and Graves (1989). Note that the MCNFP algorithms are developed for general networks, whereas our ability to solve larger problems is mainly due to the extensive use of the structural properties of UPL&FTAP throughout the solution process.

7 Concluding Remarks

In this paper, we propose an analytical approach for simultaneous optimization of plant location, capacity acquisition and technology selection in a multicommodity environment with product-dedicated and flexible technology alternatives. We modeled the problem as a mixed integer nonlinear program and suggested heuristic and exact solution methods that iteratively solve a series of mixed integer linear programs. Our computational results are quite satisfactory.

In the event that the plant locations are predetermined, UPL&FTAP reduces to a plant loading

problem. This problem focuses on the product mix at each plant as well as the technology content. The reader is referred to Cohen and Moon (1991) for a formal description of plant loading. Note that the subproblems that arise during the solution of the plant loading problem by the use of PPLU are of the form of UFLP. Thus, the exact and heuristic solution procedures presented in this paper would be quite efficient in solving this special case of UPL&FTAP.

Our analytical framework can also be used in evaluating alternative manufacturing strategies. For example, a market-focus strategy involves manufacturing all the products required by a customer zone at a single plant, which is designated to serve that area. This strategy can be evaluated by solving a series of plant loading problems each representing an alternative way of achieving the desired market focus. Alternatively, a product-focus strategy would attempt to centralize the production of each commodity so as to maximize the potential benefits from economies of scale. This strategy can be evaluated by solving a series of single-product UPL&FTAPs and compiling their solutions in designing the associated production-distribution network. Clearly, market-focus and product-focus are at the two ends of a spectrum of manufacturing strategies a company can possibly pursue. Note that, the optimal configuration prescribed by our model corresponds to the most appropriate hybrid strategy on this spectrum.

The proposed analytical framework is valuable in improving our understanding of the facility design decisions. Nevertheless, it can be enhanced in several directions: First, economies of scale in transportation could be included in the model. Second, the model could be extended to incorporate simultaneous configuration of multiple echelons of facilities i.e., subassembly plants, assembly plants, distribution centers and warehouses. Third, our model could also be extended to include possible diseconomies of scale and scope once the facilities reach a certain size. Finally, differentiating features of global manufacturing e.g., exchange rate and price uncertainties, tariffs and quotes, could be included in the model.

Appendix

Proof of Proposition 2

Let,

$$
\begin{aligned}
D_i^p &= \sum_{j\in J}(x_{ijp}^F + x_{ijp}^D), \ \ \forall (i,p), \\
D_i^f &= \sum_{q\in P\backslash p}\sum_{j\in J} x_{ijq}^F \ \ \forall i.
\end{aligned}
$$

D_i^p represents the total production of product p and D_i^f represents the production of other products by flexible technology at plant i. To prove the proposition we need to show the following (subscript i is omitted for the sake of clarity):

$$min\{f_p(D^p) + f(D^f), f(D^p + D^f)\} \leq f_p(X) + f(D^p + D^f - X), \ \ \forall\, X \in [0, D^p].$$

Let $\lambda = X/D^p$, since $f(.)$ and $f_p(.)$ are concave functions;

$$
\begin{aligned}
f_p(X) &\geq \lambda f_p(D^p) + (1-\lambda) f_p(0) \\
f(D^p + D^f - X) &\geq \lambda f(D^f) + (1-\lambda) f(D^p + D^f)
\end{aligned}
$$

Summing them over and considering $f_p(0) = 0$, we get:

$$
\begin{aligned}
f_p(X) + f(D^p + D^f - X) &\geq \lambda(f_p(D^p) + f(D^f)) + (1-\lambda) f(D^p + D^f) \\
&\geq \lambda \min\{f_p(D^p) + f(D^f), f(D^p + D^f)\} \\
&\quad +(1-\lambda)\min\{f_p(D^p) + f(D^f), f(D^p + D^f)\} \\
&= min\{f_p(D^p) + f(D^f), f(D^p + D^f)\}.
\end{aligned}
$$

Proof of Proposition 3

Upper bound: Consider the MCNFP representation: Number of extreme flows are determined by the different flow patterns between the facilities and customer-product pairs. Each customer-product pair has $2m$ arcs entering because, each plant can produce a product at a dedicated or flexible facility. In an extreme flow only one of them could be nonzero. All different combinations amount to $(2m)^{nr}$. Note that, those of combinations which violates Proposition 2 are counted as well even though they can never be optimal solution of the subproblems.

Lower bound: Let us assume that there are no flexible (dedicated) technology employed at any plant. Then the possible number of combinations reduce to m^{nr} (m^{nr}). Therefore,

$$\text{Lower bound } = m^{nr} + m^{nr} = 2m^{nr}$$

Proof of Proposition 4

Consider all pseudo-facilities, dedicated (p, k) and flexible k as warehouses, customer-product pairs, (j, p), as customers and plants, i, as distribution centers. According to notation of Gao and Robinson (1992) let i', j', k' represent indices of DCs, warehouses and customers respectively. Furthermore, if we define

$$
\begin{aligned}
F_{i'} &= F_i \\
F_{i'j'} &= \begin{cases} F_{ik} & \text{if } k \text{ is a flexible pseudo-facility of } i \\ F_{ipk} & \text{if } k \text{ is a dedicated pseudo-facility of } i \\ +\infty & \text{otherwise} \end{cases} \\
C_{i'j'k'} &= \begin{cases} C^F_{ijpk} & \text{if } k \text{ is a flexible pseudo-facility of } i \\ C^D_{ijpk} & \text{if } k \text{ is a dedicated pseudo-facility of } i \\ +\infty & \text{otherwise} \end{cases} \\
D_{k'} &= D_{jp}
\end{aligned}
$$

The resulting model is a TUFLP with (n) distribution centers, $(\sum_i k_i + \sum_i \sum_p k_{ip})$ warehouses, and (nr) customers.

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